ON THE THEORY OF THE NORMAL COMBUSTION VELOCITY

(K TEORII NORMAL'NOI SKOROSTI GORENIIA)

PMM Vol.29, № 4, 1965, pp.794-795

G.P.CHEREPANOV

(Moscow)

(Received March 30, 1964)

The normal velocity of homogeneous stationary combustion which is a physico-chemical constant of a mixture, is determined from the solution of the system of equations [1]

$$m\frac{du}{d\xi}-\frac{d^2u}{d\xi^2}=\Phi(u)\,c,\qquad m\frac{dc}{d\xi}-\lambda\frac{d^2c}{d\xi^2}=-\Phi(u)\,c \qquad (1)$$
$$(-\infty\leqslant\xi\leqslant\infty)$$

satisfying the boundary conditions

$$u(-\infty)=u_-,\qquad u(+\infty)=u_+\qquad (u_-<u_\varepsilon,\ u_+>u_\varepsilon)$$
$$c(-\infty)=v_-,\qquad c(+\infty)=0\qquad (v_->0,\ m>0) \qquad (2)$$
$$\Phi(u)=0\quad\text{for}\quad u<u_\varepsilon,\qquad \Phi(u)>0\quad\text{for}\quad u>u_\varepsilon;\ \lambda=D\gamma\rho/k$$

Here m is the normal combustion velocity; u the mixture temperature; c the concentration of active material; $c\Phi(u)$ the monomolecular reaction rate; D the diffusion coefficient; ρ the density of the material; γ its specific heat; k the coefficient of heat conduction.

For $\lambda=1$ the solution of the problem has been obtained by Zel'dovich [1]. Kanel' [2] proved the existence of the solution for all λ and the uniqueness of the solution for $0<\lambda<1$. Novikov and Riazantsev [3] investigated the problem in the singular case $\lambda=0$. An exact analytical solution of the problem (1), (2) is found below.

It is easy to reduce the original problem (1), (2) to the following. Required to find a number α from the boundary value problem on the segment [0, 1]

$$\frac{dz}{dt}=-1+\alpha f(t)\frac{v}{z},\qquad \lambda\frac{dv}{dt}=1+\frac{t-v}{z},\qquad \begin{matrix} z=0, & v=0 & \text{for}\ t=0\\ z=a & \text{for}\ t=1 & (a>0)\end{matrix} \qquad (3)$$

Here

$$f(t)=\Phi(u),\qquad f(t)=0\quad\text{for}\quad t=1,\qquad f(t)>0\quad\text{on}\ [0,1)$$

$$v=\frac{c}{u_+-u_\varepsilon},\qquad z=\frac{m^{-1}}{u_+-u_\varepsilon}\frac{du}{d\xi},\qquad t=\frac{u_+-u}{u_+-u_\varepsilon},\qquad a=\frac{u_\varepsilon-u_-}{u_+-u_\varepsilon},\qquad \alpha=\frac{1}{m^2}$$

Let us find the solution of the problem (3) by considering that the function $f(t)$ may be represented by a Taylor series whose range of convergence is greater than unity

$$f(t)=\sum_{n=0}^{\infty}f_nt^n \qquad (4)$$

In particular this may be simply a polynomial approximating the experimental curve $f(t)$.

Let us first consider the Cauchy problem for (3) with Cauchy data at $t=0$ by considering α known. Let us seek the solution of the Cauchy problem in the form

$$z = \sum_{n=1}^{\infty} z_n t^n, \qquad v = \sum_{n=1}^{\infty} v_n t^n \tag{5}$$

Substituting the functions z and v according to (5) and (3) and equating coefficients of like powers of t to zero we obtain an infinite system of algebraic equations for z_n and v_n. The first two equations of the system contain only z_1 and v_1. There exist three solutions of these equations, of which one is positive, and two negative. Only the positive solution has physical meaning

$$z_1 = \left(\frac{1}{4\lambda^2} + \alpha\frac{f_0}{\lambda}\right)^{1/2} - \frac{1}{2\lambda}, \qquad v_1 = \frac{1+z_1}{1+\lambda z_1} \tag{6}$$

The remaining equations of the infinite system are linear in the unknowns z_n and v_n. The solution of the system is expressed by the following recursion formulas:

$$z_2 = \alpha\frac{f_1 v_1(2\lambda z_1+1)}{\Delta_2}, \qquad v_2 = -\alpha\frac{f_1 v_1(\lambda v_1 - 1)}{\Delta_2}$$

$$\Delta_2 = (1+3z_1)(1+2\lambda z_1) + (\lambda-1)z_1 \tag{7}$$

. . . .

$$z_n = \frac{(1+n\lambda z_1)B_n - \alpha f_0 A_n}{\Delta_n}, \qquad v_n = -\frac{[1+(n-1)z_1]A_n + (\lambda v_1 - 1)B_n}{\Delta_n}$$

$$\Delta_n = [1+(n+1)z_1](1+n\lambda z_1) + (\lambda-1)z_1$$

$$A_n = 2\lambda v_2 z_{n-1} + 3\lambda v_3 z_{n-2} + \ldots + (n-1)\lambda v_{n-1} z_2$$

$$B_n = \alpha(f_1 v_{n-1} + f_2 v_{n-2} + \ldots + f_{n-1} v_1) - 2z_2 z_{n-1} - 3z_3 z_{n-2} - \ldots - (n-1)z_{n-1}z_2$$

It can be shown that if the range of convergence of the series (4) for the function $f(t)$ is greater than unity, then the radius of convergence of the series (5), yielding the solution of the Cauchy problem, will also be greater than unity. As is seen from the solution (5) to (7), the functions z and v are analytic functions of the variable t and the parameter α.

Knowing the solution of the Cauchy problem for arbitrary α, the value of α corresponding to the boundary condition (3) at $t=1$ should be defined as the positive root of Equation

$$\sum_{n=1}^{\infty} z_n(\alpha) = a \tag{8}$$

In particular, the following theorem results from the exposition:

T h e o r e m . The number of solutions of the original boundary value problem (3) equals the number of zeros of the function

$$\psi(\alpha) = \sum_{n=1}^{\infty} z_n(\alpha) - a$$

located on the positive real semiaxis ($z_n(\alpha)$ are defined by (7)).

It is easy to see that the function $\psi(\alpha)$ always has at least one zero on the real positive semiaxis, since, according to (7), it increases monotonously for large α and takes the value $-a$ for $\alpha = 0$.

The author is grateful to R.D.Bachelis and V.G.Melamed for useful discussions.

BIBLIOGRAPHY

1. Zel'dovich, Ia.B., K teorii rasprostraneniia plameni (On the theory of flame propagation). Zh.fiz.Khim, Vol.22, № 1, p.27, 1948.
2. Kanel', Ia.I., O statsionarnom reshenii dlia sistemy uravnenii teorii goreniia (On the stationary solution for a system of equations of combustion theory). Dokl.Akad.Nauk SSSR, Vol.149, № 2, 1963.
3. Novikov, S.S. and Riazantsev, Iu.S., K teorii goreniia kondensirovannykh sistem (On the theory of combustion of condensed systems).Dokl.Akad. Nauk SSSR, Vol.157, № 6, 1964.

Translated by M.D.F.